

Box models are a basic concept in carbon cycle models. In the lecture notes, a set of differential equations describing such box models are solved analytically. In carbon cycle modelling integrations are done numerically with the aid of mighty computers.

Consider a 1-Box model for the atmosphere that can be described with the following differential equation for a perturbation in atmospheric CO₂ ($N(t)$):

$$\frac{dN(t)}{dt} = E(t) - \frac{1}{\tau} N(t) \quad (1)$$

$N(t)$ is the atmospheric carbon (C) inventory as a function of time. $E(t)$ are time-dependent C emissions into the atmosphere. τ is a constant atmospheric life time. The discretized form of equation (1) reads

$$N_{t+1} - N_t = E_t - \frac{1}{\tau} N_t \quad (2)$$

Here, variables are not continuous in time anymore, but are updated at each time step t . E_t are thus the emissions for the duration of the time step. In the same sense, τ is in units of time step.

Write your own 1-box model in R or your programming language of preference. Use an atmospheric life time τ of 100 years. Design experiments, each spanning 1000 years, as follows:

1. Prescribe a pulse emission of 1000 GtC in year 100. Plot the perturbation of atmospheric CO₂ over time.
2. Prescribe a step change in emissions: from 0 GtC/yr before year 100 to 10 GtC/yr thereafter.
3. Prescribe linearly increasing emissions (from 0 GtC/yr in year 1 to 20 GtC/yr in year 1000). Plot atmospheric CO₂ over time.
4. Compare your results with the time scale of the decay of an atmospheric perturbation simulated by complex models as given in Fig. 1 of Problem Set 1. Discuss the caveats of the 1-Box model.